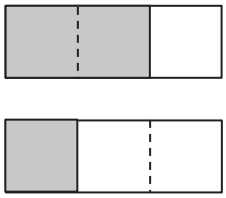
In the last two lessons we looked at fractional parts and wholes. Today we are going to go one step further in our journey toward a deep understanding of math. We are going to explore multiplying two fractions. Since the math operations are the same no matter what type of number we are working with, the path is going to be a familiar one. Here's your next "real-life adventure."

Tyler had $^{2}/_{3}$ of a watermelon. He gave $^{1}/_{2}$ of it to his brother. What part of the watermelon did his brother get?

Whenever we come across a new problem, we've learned to draw a picture of it so we can see exactly what is happening. We don't have to draw a watermelon because the drawing doesn't have to be an exact replica. A bar or block is easier to work with.



Tyler had $^{2}/_{3}$

He gave away $^{1}/_{2}$ of his $^{2}/_{3}$

Notice that because we have a proper fraction, we are dealing with a part that is less than 1. Then, with the other proper fraction, we are dealing with a part of that part. The product will always be smaller than either part.

We can see from the drawing that Tyler's brother got **$^{1}/_{3}$ of the watermelon**. Now let's see how we do the arithmetic. Looking back at the storyline, what we need to find out is, "What **is** $^{1}/_{2}$ **of** $^{2}/_{3}$?" We know that *is* means *equals* and *of* means *times*. So we can rewrite the question as, "What = $^{1}/_{2}$ **x** $^{2}/_{3}$?"
The arithmetic problem looks like this: $^{1}/_{2}$ x $^{2}/_{3}$ = ?

We can multiply two fractions in the same way we did when multiplying a whole number and a fraction. When we multiply the numerators and denominators, we get $^{2}/_{6}$. By now you should be able to recognize that the fraction is not in lowest terms since both numbers are even and divisible by 2. **From now on, your answers should always be in lowest terms.**

$$\frac{1}{2} \text{ x } \frac{2}{3} = \frac{1 \text{ x } 2}{2 \text{ x } 3} = \frac{2}{6} = \frac{1}{3}$$

The final answer **$^{1}/_{3}$** matches what we saw in the drawing.

> **Principle:** To multiply two fractions, multiply the two numerators together, and multiply the two denominators together. Reduce the fraction to lowest terms.

Let's do one more problem together: What is $^{2}/_{3}$ of $^{5}/_{6}$? Now that you've seen how to multiply fractions, we will leave out the extra step of showing the multiplication of the numerators and the denominators.

$$\frac{2}{3} \text{ x } \frac{5}{6} = \frac{10}{18} = \mathbf{\frac{5}{9}}$$

Once again you should be able to recognize that the answer of $^{10}/_{18}$ is not in lowest terms. Since both numbers are even, you know that they are divisible by 2. If you don't know for sure if the fraction is in lowest terms, use the GCD (Greatest Common Divisor) to see if it can be reduced. If you don't remember how to find the GCD, review Lesson 44.

Practice

Find the answers. Reduce all fractions to lowest terms.

(1) What is $^1/_2$ of $^1/_7$?

(2) What is $^1/_3$ of $^2/_5$?

(3) What is $^1/_3$ of $^5/_6$?

(4) What is $^2/_3$ of $^3/_5$?

(5) What is $^3/_4$ of $^2/_7$?

(6) What is $^3/_5$ of $^3/_8$?

(7) What is $^2/_5$ of $^3/_7$?

(8) What is $^3/_5$ of $^8/_9$?

(9) What is $^5/_6$ of $^4/_7$?

(10) What is $^2/_3$ of $^7/_{10}$?

(11) What is $^1/_6$ of $^1/_6$?

(12) What is $^{11}/_{12}$ of $^5/_6$?

(13) What is $^1/_4$ of $^2/_9$?

(14) What is $^1/_9$ of $^5/_7$?

(15) What is $^4/_9$ of $^2/_3$?

When we multiply a **whole number and a proper fraction**, we learned that the answer will always be a number that is **smaller than the whole number factor** (but not smaller than the proper fraction factor). Do you remember why?

When we are working with two **proper fractions**, both factors are parts of a whole. So when we multiply two proper fractions, the answer will be **a smaller fractional part than either of the factors**. Can you say why? It's because we are dealing with a part of a part.

What do you think would happen if you need to multiply an **improper fraction by a proper fraction**? For example, "What is $^6/_5$ of $^2/_3$?" Do you think the improper fraction would change the algorithm? Before you answer, think about the consistency of math.

If you said it wouldn't change the algorithm, you are right.

$$\frac{6}{5} \times \frac{2}{3} = \frac{12}{15} = \frac{4}{5}$$

An **improper fraction actually stands for a whole number and a fractional part**, so just as with a whole number times a proper fraction the product will be a number that is **smaller than the improper fraction factor** but not smaller than the proper fraction factor.

We see that if one of the two factors in a multiplication problem is a proper fraction, then the answer will be less than the other factor.

But what happens when both factors are either **whole numbers or improper fractions**?

For example, $^3/_2 \times {}^4/_3 = {}^{12}/_6$ or 2. The improper fraction $^3/_2$ is equal to $1^1/_2$, and the improper fraction $^4/_3$ is equal to $1^1/_3$. In this case the answer (2) is **larger** than each of the factors ($1^1/_2$ and $1^1/_3$).

From this example we see that if both factors are greater than or equal to one whole, their product will be greater than either one of the factors. **When you multiply two numbers that are greater than (or equal to) one, their product will be greater than each of the factors.**

Now consider this real-life situation which does involve fractional parts. *Thomas has $^3/_4$ of an apple. He wants to give his brother $^1/_2$ of what he has. What part of the whole apple would Thomas give him?*

We need to find: $^1/_2$ of $^3/_4$ is what. That's the same as $^1/_2 \times {}^3/_4 = ?$

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Thomas would give his brother **$^3/_8$ of the apple**.

We multiply any two fractions, proper or improper, in the same way. Do you think that the order of the fractions matters? If you said no, you're right.

Practice

Find the answers. Reduce all fractions to lowest terms.

(1) $^1/_5 \times {}^2/_3$? (2) $^5/_7 \times {}^2/_3$? (3) $^3/_9 \times {}^2/_3$? (4) $^4/_5 \times {}^1/_7$?

(5) $^2/_3 \times {}^7/_6$? (6) $^6/_5 \times {}^2/_9$? (7) $^1/_2 \times {}^7/_2$? (8) $^9/_8 \times {}^7/_5$?

(9) If Karen has $^1/_2$ of an orange and gives her brother $^1/_2$ of what she has, what part of an orange would she give him?

(10) Keith has $^4/_5$ of a bag of candy. If he shares $^2/_3$ of what he has with his friend Kevin, what part of the bag would Kevin get?

(11) Cynthia has $^1/_2$ bushels of apples, and gives $^1/_5$ of these apples to her aunt. What fractional part of a bushel did her aunt receive?

(12) Vic has $^2/_4$ of a cake and gives his sister, Vanessa, $^1/_3$ of it. What part of the whole cake does Vanessa have?

(13) Sam owned $^4/_5$ of a farm. He sold $^3/_7$ of his part of the farm to Charles. What part of the whole farm did Sam sell?

(14) Paige has $^3/_5$ of a box of cookies left from her lunch. Her friend Becca asked Paige to share $^1/_2$ of the box with her. What part of the box would Paige share?

(15) Two men went together to buy a boat. One of the men who owns $^7/_{10}$ of the boat wants to sell $^1/_5$ of his part to a third man. What part of the boat would he sell?